

# CHIRAL SYMMETRY AND WEAK DECAY OF HYPERNUCLEI

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The weak decays of hyperons and hypernuclei are studied from the chiral symmetry viewpoint. The soft pion relations are useful in understanding the isospin properties of the weak hyperon decays. Recent development on the short-range part of the  $\Lambda N \rightarrow NN$  weak transitions shows fairly good account of the weak decays of hypernuclei, though it fails to explain the  $n/p$  ratio. The  $\pi^+$  decays of light hypernuclei are studied in the soft pion approach. They are related to the  $\Delta I = 3/2$  amplitudes of the nonmesonic decay.

## 1 Introduction

While chiral symmetry is a powerful tool in understanding properties of low lying hadrons and their interactions, its role in the weak decays of hypernuclei has not been explored as far as we know. Here we suggest that chiral symmetry is significant in understanding weak mesonic decays of hyperons in the free space and also in nuclear medium. On the other hand, it was recently shown that the short range part of the weak  $\Lambda N \rightarrow NN$  transition is attributed to the direct quark processes, and numerical calculation suggest that it is significant in nonmesonic weak decay of hypernuclei.<sup>1,2</sup>

## 2 Chiral Symmetry

What is the role of chiral symmetry in the hyperon decay? A useful tool to take account of the chiral symmetric dynamics of hadrons is the soft pion theorem,<sup>3</sup>

$$\langle \alpha \pi^a(q) | \hat{O} | \beta \rangle \xrightarrow{q^\mu \rightarrow 0} -\frac{i}{f_\pi} \langle \alpha | [Q^a, \hat{O}] | \beta \rangle + (\text{pole terms}) \quad (1)$$

This can be applied to the weak pionic processes, such as,

$$\langle n \pi^0(q) | H^{PV} | \Lambda \rangle \xrightarrow{q^\mu \rightarrow 0} -\frac{i}{f_\pi} \langle n | [Q_5^0, H^{PV}] | \Lambda \rangle = -\frac{i}{2f_\pi} \langle n | H^{PC} | \Lambda \rangle \quad (2)$$

Here  $H^{PV}$  is the parity violating part of the weak hamiltonian, which contains only the left-handed currents  $\bar{q}_L^a \gamma^\mu q_L^a$  and the flavor singlet right handed current induced by the penguin type QCD corrections. This allows us to relate

the commutation  $[Q_5^0, H^{PV}]$  in the second expression of eq.(2) to  $H^{PC}$  in the final expression as

$$\begin{aligned} [Q_R^a, H_W] &= 0 \\ [Q_5^a, H_W] &= -[Q_L^a, H_W] = -[I^a, H_W] \\ \langle n|[Q_5^a, H^{PV}]|\Lambda\rangle &= \langle n|[I^a, H^{PC}]|\Lambda\rangle = -\frac{1}{2}\langle n|H^{PC}|\Lambda\rangle \end{aligned}$$

Thus, the parity violating amplitudes, or the  $S$ -wave decay amplitudes, of pionic decays of various hyperons can be expressed in terms of the baryonic matrix elements of the parity conserving weak hamiltonian. As a result, for instance, the  $\Delta I = 1/2$  dominance of  $\Lambda \rightarrow N\pi$  decays follows immediately since “ $\Lambda \rightarrow n$  transition” is purely  $\Delta I = 1/2$ . Furthermore if we conjecture that  $H_W$  is purely flavor octet, then various matrix elements,  $\langle n|H^{PC}|\Sigma\rangle$ ,  $\langle \Sigma|H^{PC}|\Xi\rangle$ , are all related to  $\langle n|H^{PC}|\Lambda\rangle$  and thus several relations among the pionic decay amplitudes of hyperons are obtained. Such relations are known to be satisfied fairly well for the  $S$ -wave decay amplitudes of the hyperon decays.<sup>4</sup>

The parity conserving decays belong to exceptions of the soft pion theorem, in which the pole terms cannot be neglected. The pole terms are such that  $\Lambda \rightarrow n \rightarrow n\pi^0$  or  $\Lambda \rightarrow \Sigma^0 \pi^0 \rightarrow n\pi^0$ , and their amplitudes are

$$\begin{aligned} \langle n\pi^0(q \rightarrow 0)|H^{PC}|\Lambda\rangle &\sim \langle n\pi^0|n\rangle \frac{i}{m_\Lambda - m_n} \langle n|H^{PC}|\Lambda\rangle \\ &+ \langle n|H^{PC}|\Sigma^0\rangle \frac{i}{m_n - m_\Sigma} \langle \Sigma^0\pi^0|\Lambda\rangle \end{aligned} \quad (3)$$

Assuming the pole dominance of the parity conserving,  $P$ -wave, decay amplitudes, we find interesting relations of the  $\Sigma^+ \rightarrow n\pi^+$  decay. It can be easily shown that the soft-pion amplitude vanishes,

$$\langle n\pi^+|H^{PV}|\Sigma^+\rangle_{\text{soft-pion}} = 0 \quad (4)$$

since

$$[I^-, H^{PC}] = 0 \quad (5)$$

for the  $\Delta I = 1/2$  dominant  $H^{PC}$ , which has  $\Delta I_3 = -1/2$ . On the other hand, there is no such constraint for the pole terms of the parity conserving amplitudes, where the  $p$ ,  $\Sigma^0$  and  $\Lambda$  intermediate states with different energy denominators contribute. Therefore the soft-pion theorem suggests that the  $\Sigma^+ \rightarrow n\pi^+$  decay goes only through the parity conserving  $P$ -wave channel. This is indeed what we observe experimentally, the PV amplitude 0.13 *v.s.* the PC 42.2. This example shows that the  $\Delta I = 1/2$  dominance and the soft-pion relation is very well satisfied in this decay.

We later consider  $\pi^+$  decays of hypernuclei in this context and see that the soft-pion theorem suggests the  $\pi^+$  decays are induced only by the  $\Delta I = 3/2$  part of the weak hamiltonian.

### 3 $\Delta I = 1/2$ Rule

In the above discussion, we have assumed that  $\Delta I = 1/2$  dominance of the weak matrix element,  $\langle N | H^{PC} | \Sigma \rangle$ . Explanation of the “ $\Delta I = 1/2$  rule” has been a long standing problem of the weak decay of the kaons and the hyperons. It was shown long time ago that the perturbative QCD corrections to the standard model weak vertex enhance the  $\Delta I = 1/2$  component, while it suppresses the counterpart,  $\Delta I = 3/2$  component. The mechanism can be understood easily by decomposing the weak  $s + d \rightarrow u + d$  transition into the isospin-spin-color eigenstates. As the strangeness changing transition is induced only by the charged current, or the  $W$ -boson exchange, the vertex at low energy is given without the QCD corrections by

$$(\bar{u}_L^\alpha \gamma^\mu s_L^\alpha)(\bar{d}_L^\beta \gamma_\mu u_L^\beta) = (\bar{d}_L^\beta \gamma^\mu s_L^\alpha)(\bar{u}_L^\alpha \gamma_\mu u_L^\beta) \quad (6)$$

where  $\alpha$  and  $\beta$  are color indices and the equality comes from the Fierz transformation. From this we observe that the color+isospin combination of the final  $u + d$  quarks is always symmetric, namely,  $(I_f = 0, \text{Color } \bar{\mathbf{3}})$  or  $(I_f = 1, \text{Color } \mathbf{6})$ . In both cases, the total spin of the final quarks must be 0. When we consider gluon corrections to this vertex, we notice that the gluon exchange, or its color-magnetic component,  $-(\lambda_1 \cdot \lambda_2)(\sigma_1 \cdot \sigma_2)$  term, between the final  $u$  and  $d$  is attractive for  $(I_f = 0, \text{Color } \bar{\mathbf{3}})$ ,

$$\langle S = 0, C = \bar{\mathbf{3}} | (\lambda_1 \cdot \lambda_2)(\sigma_1 \cdot \sigma_2) | S = 0, C = \bar{\mathbf{3}} \rangle = -8, \quad (7)$$

while it is repulsive in the other,

$$\langle S = 0, C = \mathbf{6} | (\lambda_1 \cdot \lambda_2)(\sigma_1 \cdot \sigma_2) | S = 0, C = \mathbf{6} \rangle = +4. \quad (8)$$

Therefore the QCD correction tends to enhance the final  $I_f = 0$  amplitude. The above heuristic explanation of the  $\Delta I = 1/2$  enhancement can be confirmed in the renormalization group improved effective action of the strangeness-changing weak interaction.<sup>5</sup> It was also shown that a further enhancement of  $\Delta I = 1/2$  is resulted due to the “Penguin” diagrams, which is regarded as a QCD-corrected  $s \rightarrow d$  transition, and is purely  $\Delta I = 1/2$ .<sup>5</sup>

The perturbative QCD corrections are not the only source of the enhancement, but we expect further effects due to nonperturbative origin. In fact, it is known that the enhancement in the effective interaction is not large enough

to explain the observed dominance of  $\Delta I = 1/2$ . We here concentrate on the baryonic weak interaction. Miura-Minamikawa and Pati-Woo<sup>6</sup> pointed out that the  $\Delta I = 3/2$  part of the nonleptonic hyperon decays is suppressed due to the color symmetry of the valence quarks in the baryon. It is understood easily by considering the color structure of the 4-quark operator which belongs to the 27-dimensional irrep. of the flavor  $SU(3)$ :

$$O(27) = (\bar{u}_L^\alpha \gamma^\mu s_L^\alpha)(\bar{d}_L^\beta \gamma_\mu u_L^\beta) + (\bar{d}_L^\alpha \gamma^\mu s_L^\alpha)(\bar{u}_L^\beta \gamma_\mu u_L^\beta) \quad (9)$$

which is the part responsible for the  $\Delta I = 3/2$  transition. From the symmetry of the final  $u$  and  $d$  quarks, this operator creates two quarks with their color part being symmetric, *i.e.*, the color **6** state. As the color wave function for the valence three quarks of the baryons, either the hyperon in the initial state or the nucleon in the final state, this operator cannot be connected to two quarks inside the baryon. Thus, the only possibility is the external diagram in which both the quark and the antiquark of the meson (pion) are connected to the weak vertex directly. Such diagram is not allowed either because the external  $q\bar{q}$  created by eq.(9) should be both left-handed and therefore does not make a pseudoscalar meson in the chiral limit. Gluon exchanges among the initial or final quarks do not help, while “exotic” component such as valence gluon will change the situation. Thus we observe that the  $\Delta I = 3/2$  part of the  $Y \rightarrow N + \text{PS meson}$  is strongly suppressed as far as we consider the valence quark picture of the baryons. It should be noted, however, that the vector mesons may couple with  $\Delta I = 3/2$  vertices directly.<sup>7</sup>

Another possibility is the  $\Delta I = 1/2$  enhancement due to the diquark components in the baryon. It is an enhancement of  $0^+ \text{ } ud(I=0)$  diquark components, which is favored by the gluon exchanges. This is in fact the same mechanism as the  $\Delta I = 1/2$  enhancement in the perturbative correction discussed above. We, however, have found that such effect may be small because the naive evaluation of the  $H^{PC}$  in the harmonic oscillator valence quark model gives strong enough transition amplitude for the  $\Lambda \rightarrow N\pi$  decay according to the soft-pion formula, eq.(2).

#### 4 Direct Quark Mechanism and Weak Decay of Hypernuclei

The pionic decay of  $\Lambda$  is known to be suppressed in nuclear medium as the final nucleon do not have enough momentum to go above the Fermi energy. Thus the main decay mode is nonmesonic, which can be described by the simplest elementary processes,  $\Lambda p \rightarrow pn$  and  $\Lambda n \rightarrow nn$ . These processes are viewed as weak baryonic interactions, which is unique and interesting itself as a new type of the nonleptonic weak interactions of baryons. Furthermore, this is a reaction

in which the momentum transfer is so large that the quark substructures of the baryons may be significant.

Recently, we proposed the direct quark (DQ) transition mechanism to account for the short-range part of the  $YN \rightarrow NN$  weak interaction.<sup>1,8</sup> The DQ transition potential is obtained by evaluating  $su \rightarrow ud$  and  $sd \rightarrow dd$  transitions among the valence quarks in two baryons.

The decay rates of the  $\Lambda$  in nuclear matter, and in light hypernuclei are calculated with the transition potential, which includes DQ and  $\pi$  and  $K$  meson exchanges. The results are compared with those without DQ, and also with experiment. We leave the details to literature,<sup>9</sup> while the conclusions of our study are summarized here. (1) The DQ transition is significantly large, and shows qualitative differences from the meson exchanges. (2) The one pion exchange (OPE) mechanism yields a large tensor amplitude, *i.e.*, the transition from  $\Lambda p :^3 S_1$  to  $np :^3 D_1$ . This seems in fact too large if we employ a hard form factor as is used in the Bonn potential. We have pointed out that softer form factors for the pion-nucleon couplings are more appropriate. (3) The large tensor amplitude in OPE causes a difficulty that the  $nn/np$  ratio is too small compared to the value suggested from experiment, so-called the  $n/p$  ratio problem. We found that this problem is not solved completely only by the introduction of a soft form factor for OPE, but it is clear that the short-range mechanisms of the weak transition are important. Indeed, the DQ amplitude enhances the  $nn$  decay rate significantly and therefore improves the ratio. (4) The  $\Delta I = 3/2$  contribution is significant for the  $J = 0$  transition amplitudes in the DQ mechanism, while the meson exchanges are assumed purely in the  $\Delta I = 1/2$  transition. It should be noted here that the mechanisms to enhance the  $\Delta I = 1/2$  amplitudes in other hadronic weak interactions are not effective here and therefore the  $\Delta I = 3/2$  amplitudes may be comparable to the  $\Delta I = 1/2$  one. Unfortunately, it is not possible at present to determine the importance of the  $\Delta I = 3/2$  contribution from the experimental data.<sup>10</sup>

## 5 $\pi^+$ decay mode and $\Delta I = 3/2$ amplitudes

Light hypernuclei may decay weakly by emitting a pion. While the free  $\Lambda$  decays into  $p\pi^-$  or  $n\pi^0$ , the  $\pi^+$  decay requires an assistance of a proton, *i.e.*,  $\Lambda + p \rightarrow n + n + \pi^+$ . Some old experimental data suggest that the ratio of  $\pi^+$  and  $\pi^-$  emission from  ${}^4_\Lambda\text{He}$  is about 5%.<sup>11</sup> The most natural explanation of this process is  $\Lambda \rightarrow n\pi^0$  decay followed by  $\pi^0 p \rightarrow \pi^+ n$  charge exchange reaction. It was evaluated for realistic hypernuclear wave functions and found to explain only 1.2% for the  $\pi^+/\pi^-$  ratio.<sup>12</sup> Another possibility is to consider  $\Sigma^+ \rightarrow \pi^+ n$  decay after the conversion  $\Lambda p \rightarrow \Sigma^+ n$  by the strong interaction

such as pion or kaon exchanges. It was found, however, that the free  $\Sigma^+$  decay which is dominated by  $P$ -wave amplitude, gives at most 0.2% for the  $\pi^+/\pi^-$  ratio.

In order to solve this problem, we have applied the soft-pion technique to the  $\pi^+$  decay of light hypernuclei.<sup>13</sup> The soft-pion theorem to the process  $\Lambda p \rightarrow nn\pi^+(q \rightarrow 0)$  reads

$$\lim_{q \rightarrow 0} \langle nn\pi^+(q) | H_W | \Lambda p \rangle = -\frac{i}{\sqrt{2}f_\pi} \langle nn | [Q_5^-, H_W] | \Lambda p \rangle \quad (10)$$

Again, because of

$$[Q_5^-, H_W] = -[I_-, H_W] \quad (11)$$

it discriminates the isospin properties of  $H_W$ . Similarly to the case of  $\Sigma^+$  decay, we see that the  $\Delta I = 1/2$  part vanishes as

$$[I_-, H_W(\Delta I = 1/2, \Delta I_z = -1/2)] = 0 \quad (12)$$

$$[I_-, H_W(\Delta I = 3/2, \Delta I_z = -1/2)] = \sqrt{3}H_W(\Delta I = 3/2, \Delta I_3 = -3/2) \quad (13)$$

We then obtain

$$\lim_{q \rightarrow 0} \langle nn\pi^+(q) | H_W | \Lambda p \rangle = \frac{i\sqrt{3}}{\sqrt{2}f_\pi} \langle nn | H_W(\Delta I = 3/2, \Delta I_3 = -3/2) | \Lambda p \rangle \quad (14)$$

Thus we conclude that the soft  $\pi^+$  emission in the  $\Lambda$  decay in hypernuclei is caused only by the  $\Delta I = 3/2$  component of the strangeness changing weak hamiltonian. In other words, the  $\pi^+$  emission from hypernuclei probes the  $\Delta I = 3/2$  transition of  $\Lambda N \rightarrow NN$ .

## 6 Conclusion

In this article, we have tried to demonstrate how the soft-pion approach is useful in understanding the weak hyperon transitions. It is amazing that the chiral symmetry plays so important role even in the weak processes. We, however, have further remaining problems. It is necessary to go beyond the chiral limit, so that the finite pion mass effects are to be included. Corrections due to the flavor  $SU(3)$  breaking may also be important. These can be included by the chiral perturbation theory approach. In view of the qualitative success in the soft-pion approach, it is promising to apply the chiral effective theories to the hyperon and hypernuclear decay processes. Several recent papers attempt such approaches for the hyperon decays with some success.<sup>14</sup> It is interesting to study the hypernuclear decays in such formulations.

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